Exercise VII

1. Prove that there is a real number L such that $L^3 = 2$.

2. Which one of the following functions is continuous at the indicated point?

   (i) $f(x) = \dfrac{\sin(x)}{x}$ at $x = 0$.

   (ii) $f(x) = \begin{cases} \dfrac{\sin(x)}{x}, & x \neq 0 \\ 1, & x = 0. \end{cases}$ at $x = 0$

   (iii) $f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3}, & x \neq 3 \\ 7, & x = 3. \end{cases}$ at $x = 3$.

   (iv) $f(x) = \begin{cases} \dfrac{1}{x}, & x \neq 0 \\ 2, & x = 0. \end{cases}$ at $x = 0$.

3. Use the properties of limits to prove carefully that the function

$$f(x) = \frac{1}{x^2 + 4}$$

   is continuous at every $x \in \mathbb{R}$.

4. Prove carefully that the function $f(x) = |x|$ is continuous at every $x \in \mathbb{R}$. Note that you will have to pay special attention to the point $x = 0$ and use left hand and right hand limits at that point.

5. Investigate continuity of the following functions

   (i) $f(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 2 - x, & 1 < x \leq 2 \end{cases}$.

   (ii) $f(x) = \begin{cases} x^2, & 0 \leq x \leq 1 \\ 2 - x, & 1 < x \leq 2. \end{cases}$